Mediation proportion

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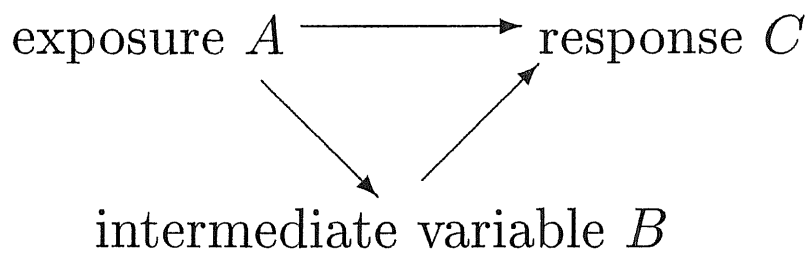
Mediation proportion

joint work with Susanne Ditlevsen (Biostat., Copenhagen), Ulla Christensen and Pernille Due (Social Medicine, Copenhagen), John Lynch (Epidemiology, Univ. of Michigan)



How much of the effect of A on C is mediated through B?

Simple regression analysis

$$\begin{array}{l}\text{exposure}\\ \text{mediator}\\ \text{response}\end{array}\begin{pmatrix} A \\ B \\ C \end{pmatrix} \text{ normal } \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{pmatrix}$$

Without B

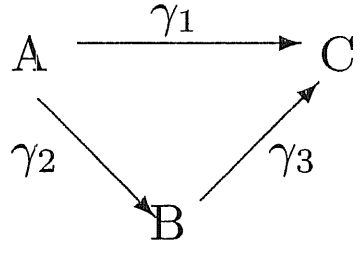
Effect of A on C: regression $C = \beta A + \varepsilon$, $\beta = \sigma_{13}/\sigma_1^2$

With B

Effect of A on B: regression $B = \gamma_2 A + \varepsilon_1$, $\gamma_2 = \sigma_{12}/\sigma_1^2$

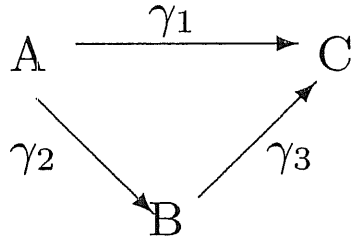
Effect of A and B on C:

regression $C = \gamma_1 A + \gamma_3 B + \varepsilon_2 = \gamma_1 A + \gamma_3\gamma_2 A + \varepsilon$



γ_1	+	$\gamma_3\gamma_2$	=	β
direct	+	indirect	=	total

Mediation proportion



$$\gamma_1 + \gamma_3\gamma_2 = \beta$$

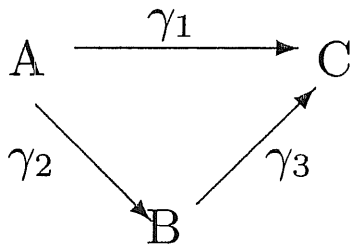
$$\text{direct} + \text{indirect} = \text{total}$$

$$\frac{\text{indirect effect}}{\text{total effect}} = \frac{\gamma_2\gamma_3}{\gamma_1 + \gamma_2\gamma_3} = \frac{\gamma_2\gamma_3}{\beta} = 1 - \frac{\gamma_1}{\beta}$$

Non-normal distribution: structural equations

Example. Effect of socioeconomic status on health mediated through hostility.

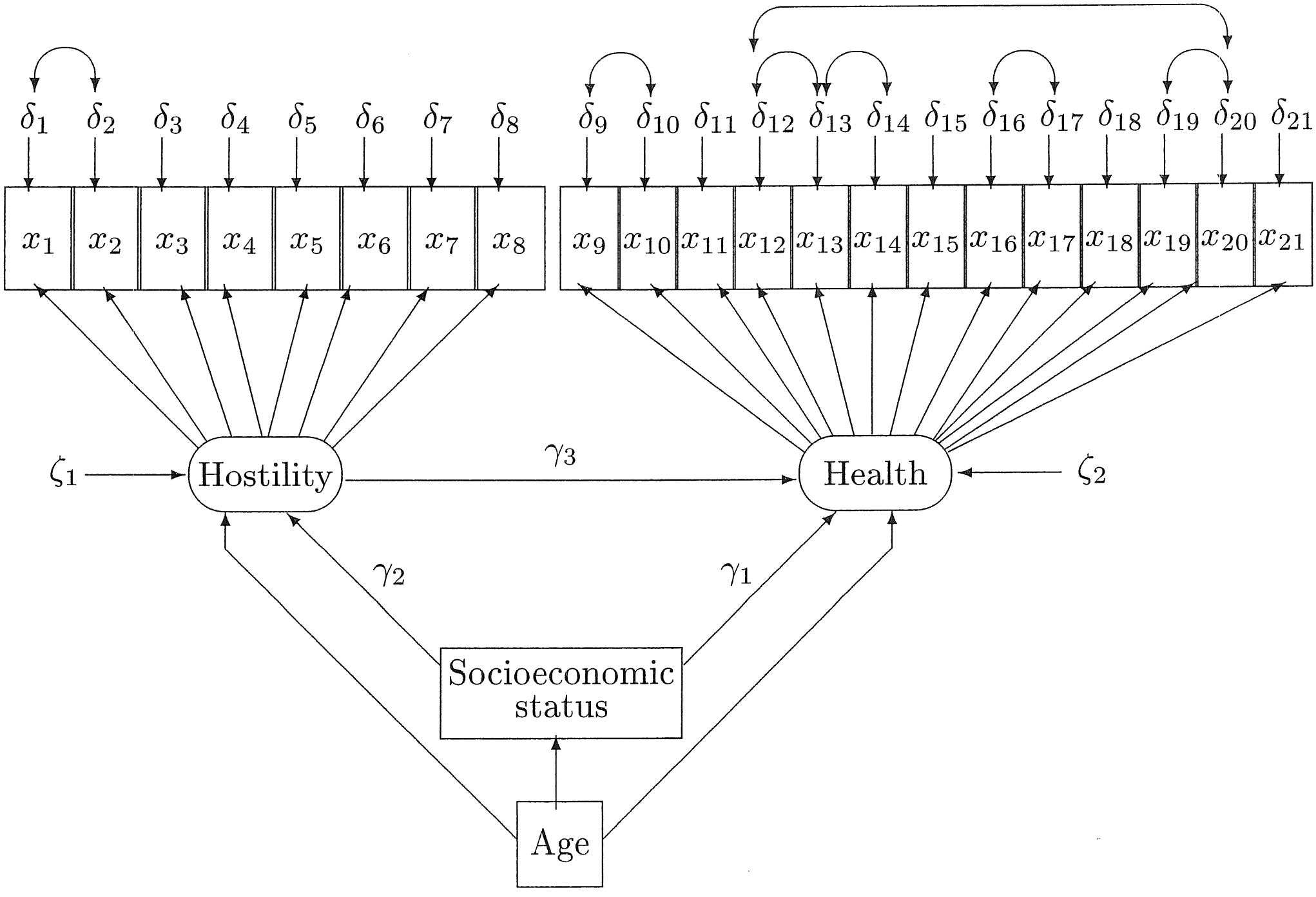
Data. Random sample of 40 and 50 years old Danes.
7588 (response proportion 69%) answered questionnaire



A Socioeconomic status 6 categories

B Hostility 8 items

C Health 13 items re symptoms during last 4 weeks


δ_1 δ_2 δ_3 δ_4 δ_5 δ_6 δ_7 δ_8
δ_9 δ_{10} δ_{11} δ_{12} δ_{13} δ_{14} δ_{15} δ_{16} δ_{17} δ_{18} δ_{19} δ_{20} δ_{21}
x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8
x_9 x_{10} x_{11} x_{12} x_{13} x_{14} x_{15} x_{16} x_{17} x_{18} x_{19} x_{20} x_{21}
ζ_1
Hostility
γ_3
Health
ζ_2
γ_2
γ_1
Socioeconomic status
Age

Param	Estimate	Std. err.	Approx. 95% CI	p-value
γ_1	0.121	0.011	$[0.099\,;0.143]$	< 0.0001
γ_2	0.207	0.013	$[0.182\,;0.232]$	< 0.0001
γ_3	0.210	0.021	$[0.169\,;0.251]$	< 0.0001

Regression estimates for men

$$\frac{\text{Indirect effect}}{\text{Total effect}} = \frac{0.210 \cdot 0.207}{0.121 + 0.210 \cdot 0.207} = 0.264 \quad (0.203; 0.325)$$

Param	Estimate	Std. err.	Approx. 95% CI	p-value
γ_1	0.165	0.012	[0.141 ; 0.189]	<0.0001
γ_2	0.193	0.013	[0.168 ; 0.218]	<0.0001
γ_3	0.234	0.023	[0.189 ; 0.279]	<0.0001

Regression estimates for women

$$\frac{\text{Indirect effect}}{\text{Total effect}} = \frac{0.234 \cdot 0.193}{0.165 + 0.234 \cdot 0.193} = 0.215 \quad (0.166; 0.264)$$